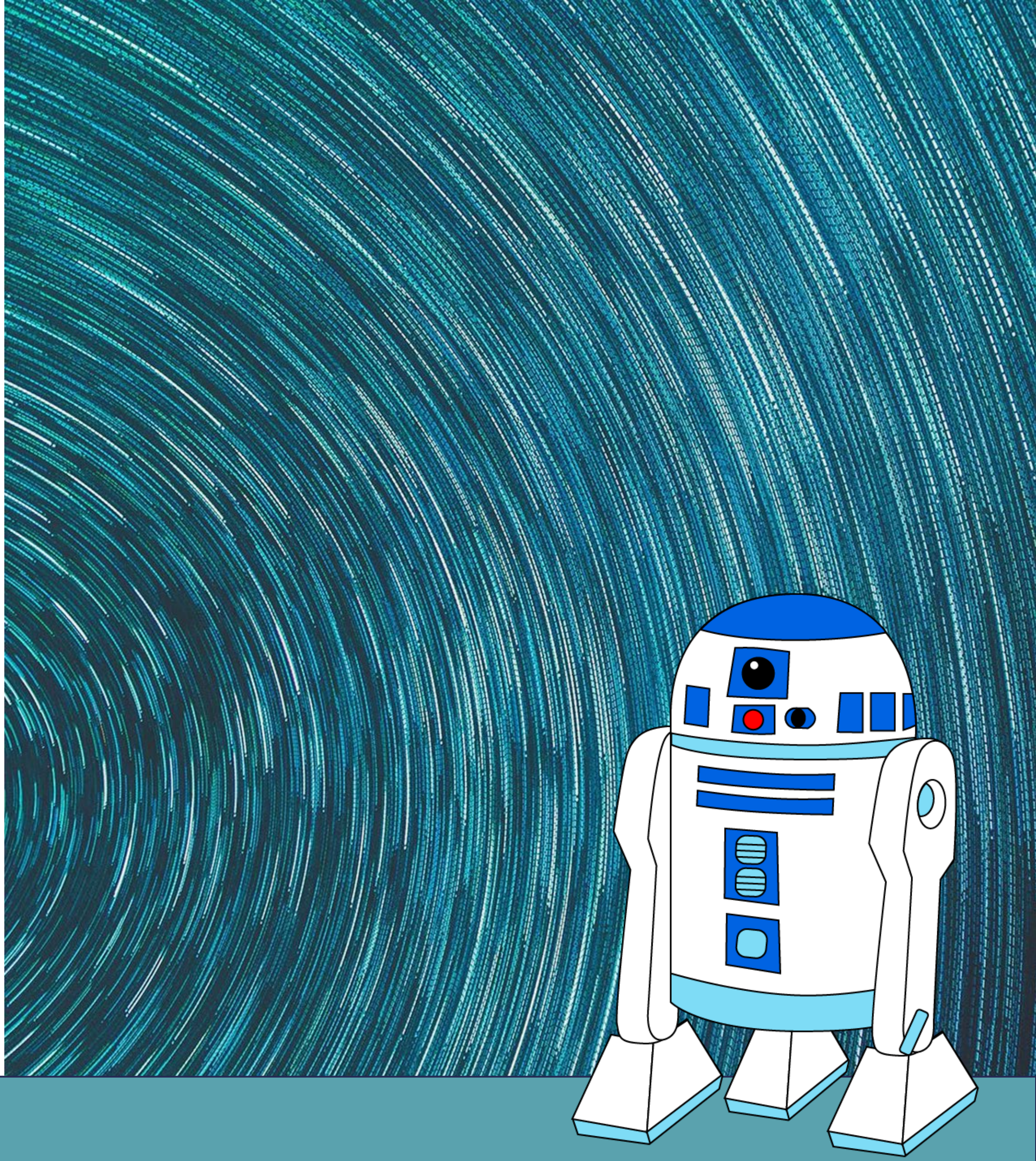
CIS 421/521:
ARTIFICIAL INTELLIGENCE

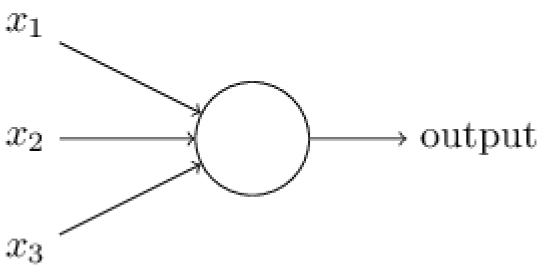
# Perceptrons

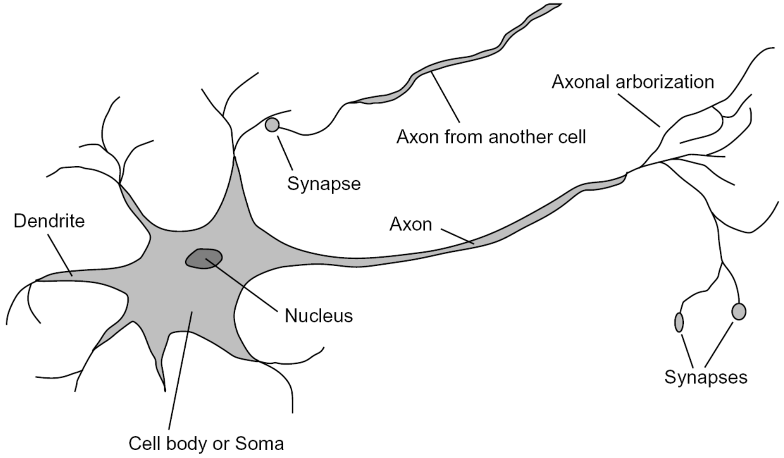


Penn Engineering
UNIVERSITY of PENNSYLVANIA

# Perceptrons

- Perceptrons were developed in the 1950s and 1960s loosely inspired by the neuron.




## Electronic 'Brain' Teaches Itself

The Navy last week demonstrated the embryo of an electronic computer named the Perceptron which, when completed in about a year, is expected to be the first non-living mechanism able to "perceive, recognize and identify its surroundings without human training or control." Navy officers demonstrating a preliminary form of the device in Washington said they hesitated to call it a machine because it is so much like a "human being without life."

Dr. Frank Rosenblatt, research psychologist at the Cornell Aeronautical Laboratory, Inc., Buffalo, N. Y., designer of the Perceptron, conducted the demonstration. The machine, he said, would be the first electronic device to think as the human brain. Like humans, Perceptron will make mistakes at first, "but it will grow wiser as it gains experience," he said.

The first Perceptron, to cost about $100,000, will have about 1,000 electronic "association cells" receiving electrical impulses from an eyelike scanning device with 400 photocells. The human brain has ten billion responsive cells, including 100,000,000 connections with the eye.

### Difference Recognized

recognize the difference between right and left, almost the way a child learns.

When fully developed, the Perceptron will be designed to remember images and information it has perceived itself, whereas ordinary computers remember only what is fed into them on punch cards or magnetic tape.

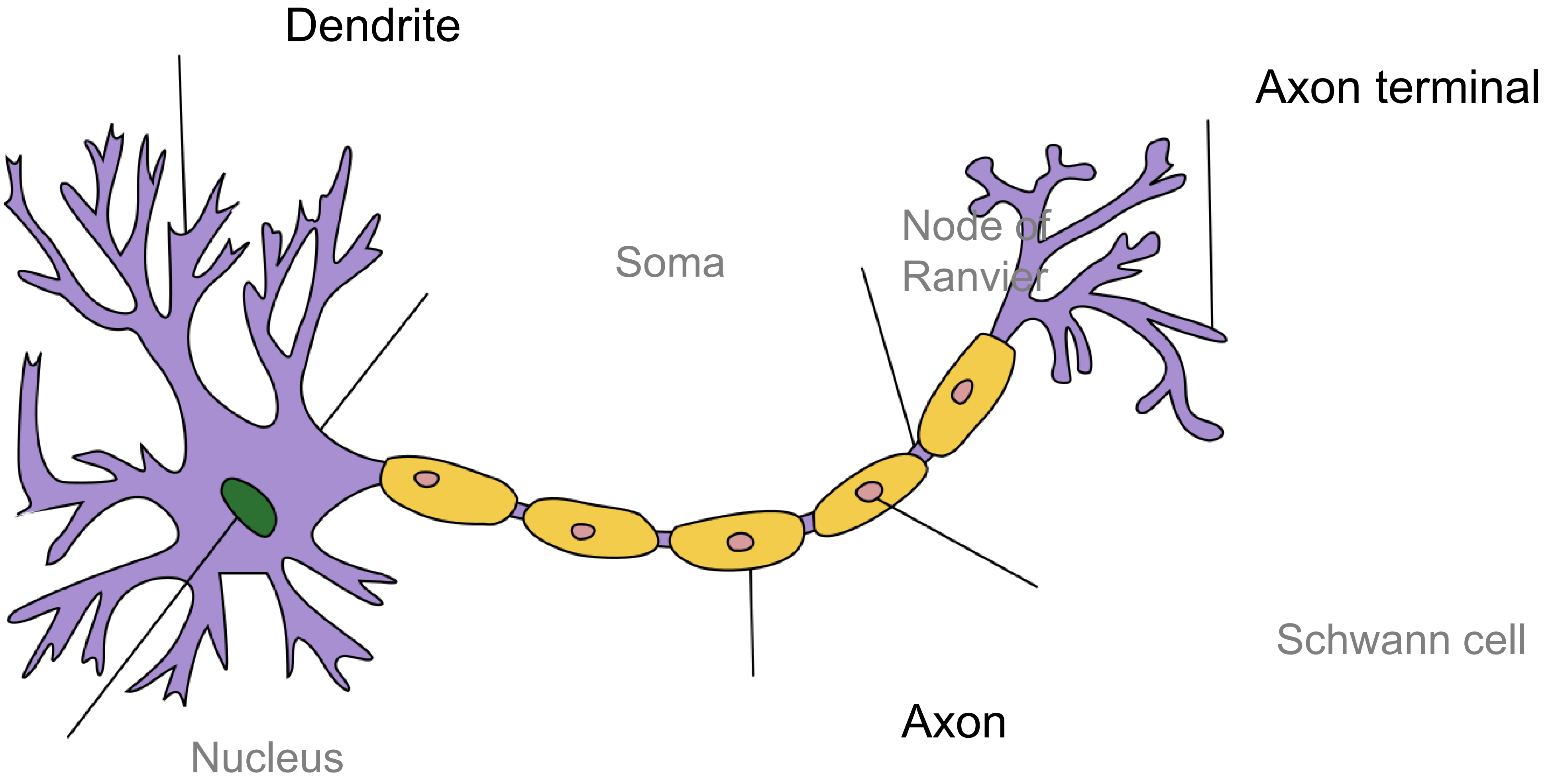
Later Perceptrons, Dr. Rosenblatt said, will be able to recognize people and call out their names. Printed pages, longhand letters and even speech commands are within its reach. Only one more step of development, a difficult step, he said, is needed for the device to hear speech in one language and instantly translate it to speech or writing in another language.

### Self-Reproduction

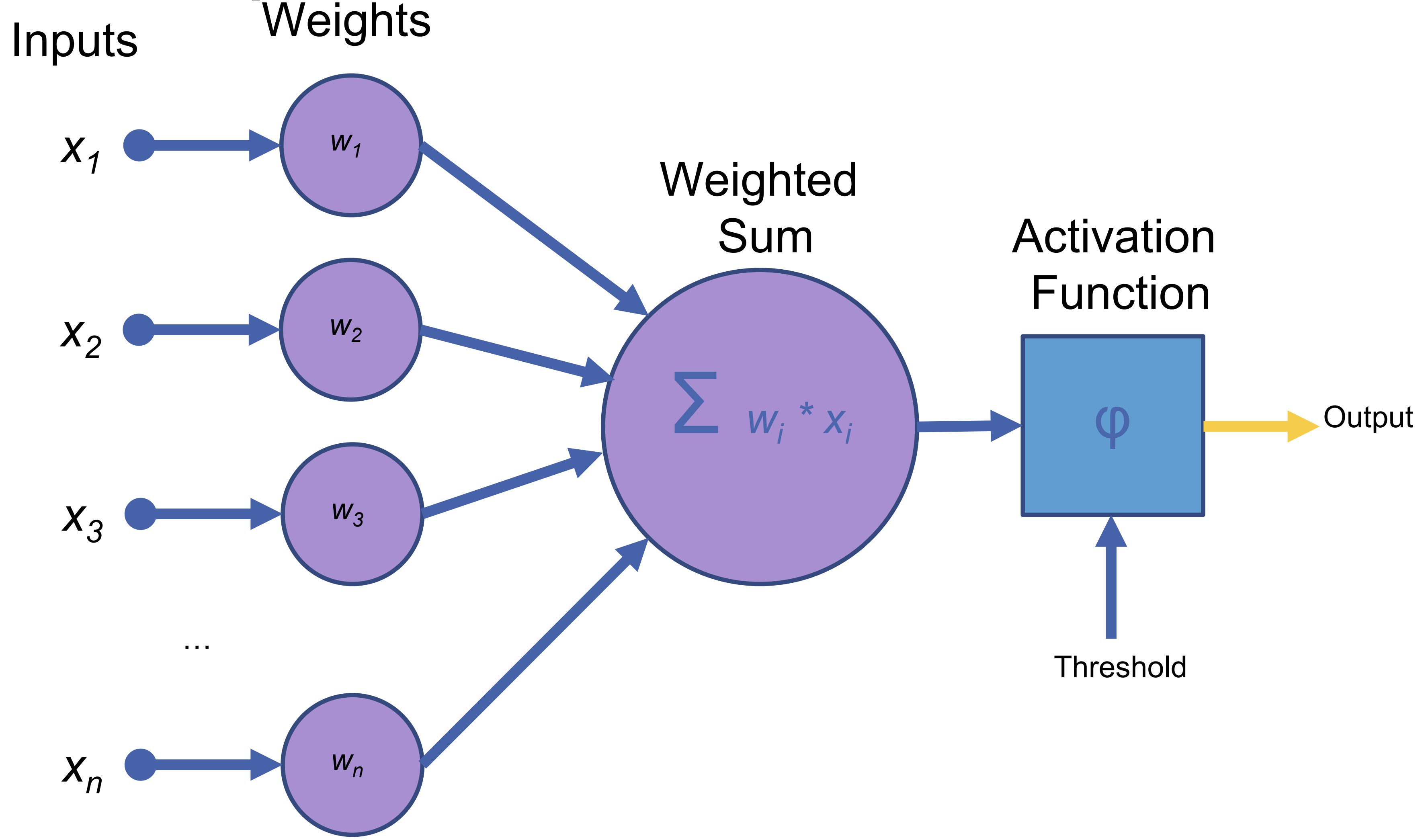
In principle, Dr. Rosenblatt said, it would be possible to build Perceptrons that could reproduce themselves on an assembly line and which would be "conscious" of their existence.

Perceptron, it was pointed out, needs no "priming." It is not necessary to introduce it to surroundings and circumstances, record the data involved and then store them for future comparison as is the case

# Neuron

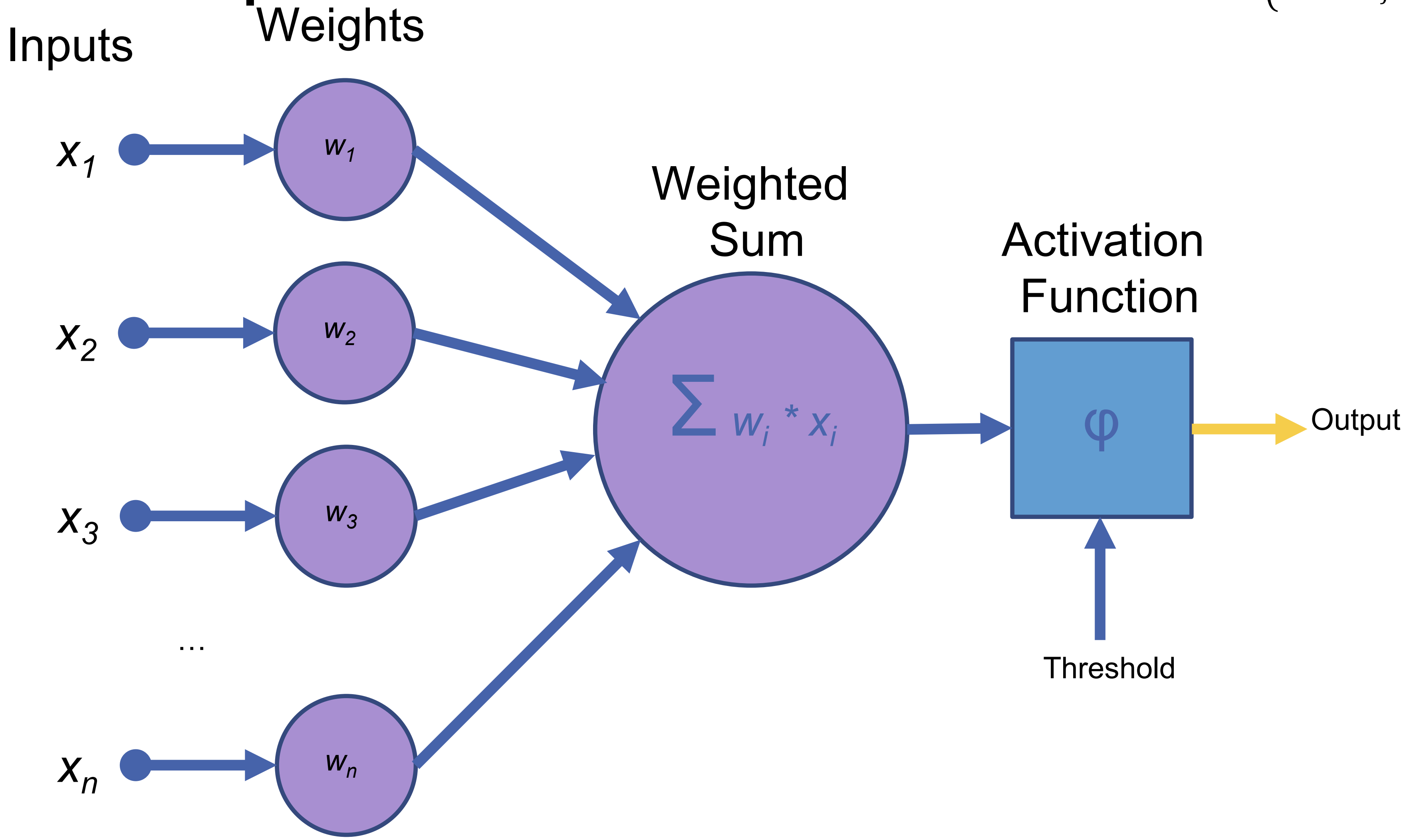

Dendrite
Axon terminal
Soma
Node of Ranvier
Schwann cell
Axon
Nucleus

# Perceptron


Inputs
Weights
$x_1$
$x_2$
$x_3$
…
$x_n$
$w_1$
$w_2$
$w_3$
$w_n$
Weighted Sum
$\sum w_i * x_i$
Activation Function
φ
Output
Threshold

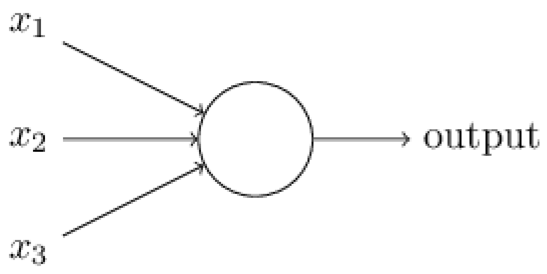
# Perceptron

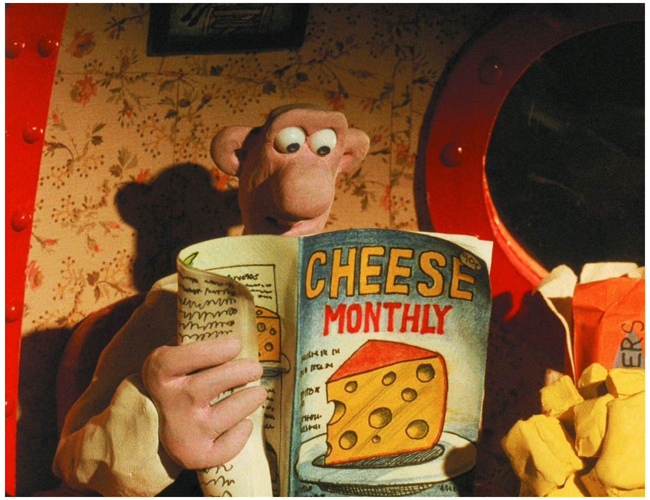
$$\text{output} = \begin{cases} 0 \ \text{if} \ \sum_j w_j x_j \leq \text{threshold} \\ 1 \ \text{if} \sum_j w_j x_j > \text{threshold} \end{cases}$$



# Perceptrons for decision making

- We can think about the perceptron or the sigmoid neuron as a device that makes decisions by weighing up evidence.
- Example: Suppose there's a cheese festival in your town.  You like cheese.




Example from Michael Nielsen's book [Neural Networks and Deep Learning]

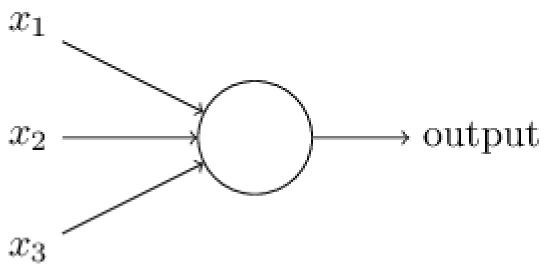
# Perceptrons for decision making

- You might use 3 factors to decide whether to go.

1. Is the weather good?
2. Can your loyal companion come with you?
3. Is the festival near public transit?

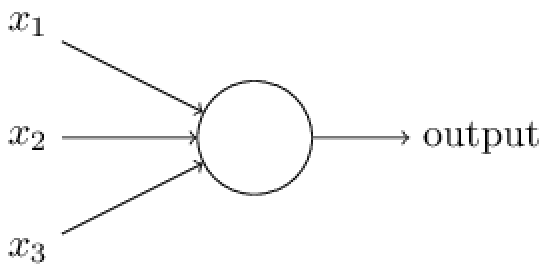
- These can be the binary input values to a perceptron



# Perceptrons for decision making

- By varying weights and the threshold we get different models of decision making
- Example 1: $w_1 = 6 \quad w_2 = 2 \quad w_3 = 2$, threshold = 5
- Example 2: $w_1 = 6 \quad w_2 = 2 \quad w_3 = 2$, threshold = 3



# Notational changes

- Change 1: We can write $\sum_j w_j x_j$ as a dot product of the input vector and the weight vector:

- $\sum_j w_j x_j \equiv \boldsymbol{w} \cdot \boldsymbol{x}$

- Change 2: We can move the threshold to other other side of the inequality. We define a perceptron's "bias" as the -1 * its threshold:

$$\text{output} = \begin{cases} 0 \ \text{ if } \sum_j w_j x_j \leq \text{threshold} \\ 1 \ \text{ if } \sum_j w_j x_j > \text{threshold} \end{cases}$$

- $b \equiv -\text{threshold}$

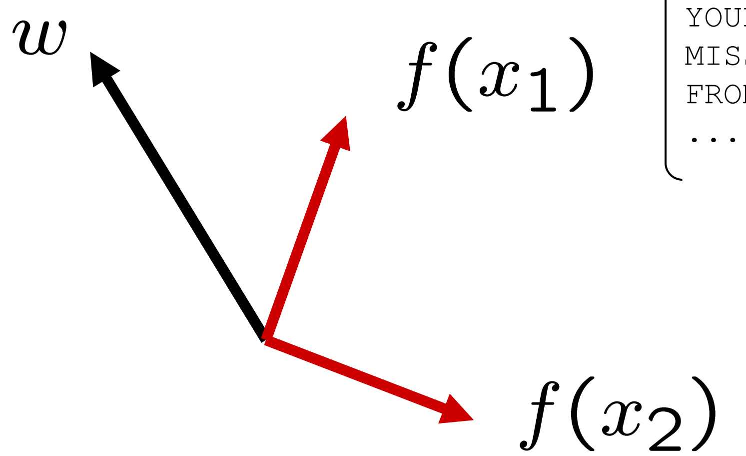
rewrites to

$$\text{output} = \begin{cases} 0 \ \text{ if } \boldsymbol{w} \cdot \boldsymbol{x} + b \ \leq 0 \\ 1 \ \text{ if } \boldsymbol{w} \cdot \boldsymbol{x} + b \ > 0 \end{cases}$$

# Learning weights from examples

- Perceptions can be used for all kinds of classification problems.
- Think of the inputs as *features* representing something we want to classify.
- The feature values for inputs are fixed, but we can choose different weight vectors. Depending on the weight vector that we pick, we will get a different classifier.

```
# free        : 4
YOUR_NAME     :-1
MISSPELLED    : 1
FROM_FRIEND   :-3
...
```

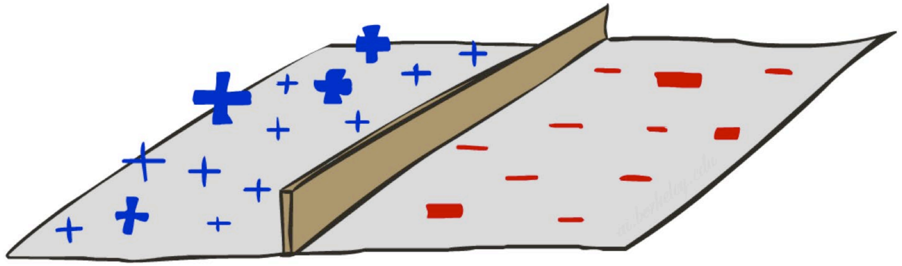



```
# free        : 2
YOUR_NAME     : 0
MISSPELLED    : 2
FROM_FRIEND   : 0
...
```

```
# free        : 0
YOUR_NAME     : 1
MISSPELLED    : 1
FROM_FRIEND   : 1
...
```

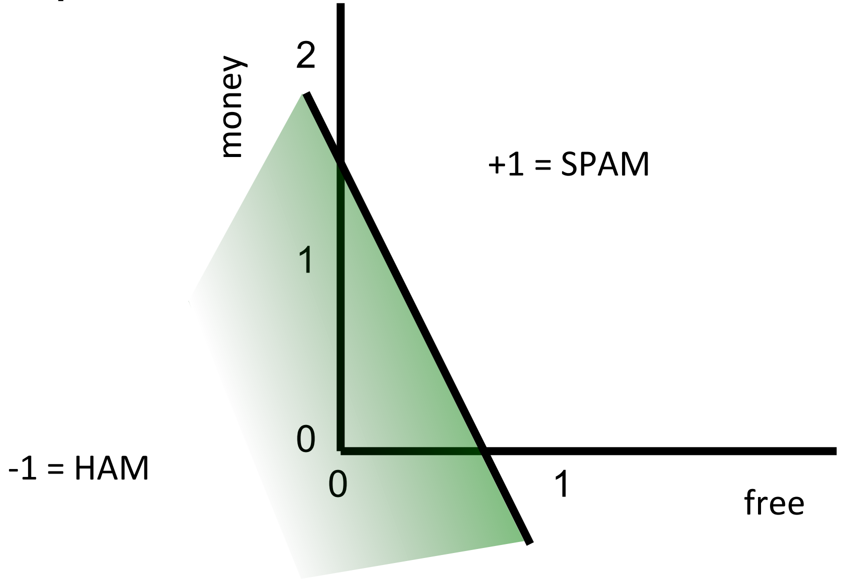
# Binary Decision Rule

- In the space of feature vectors
  - Examples are points
  - Any weight vector is a hyperplane
  - One side corresponds to Y=+1
  - Other corresponds to Y=-1



$$w$$

```
BIAS  : -3
free  :  4
money :  2
...
```

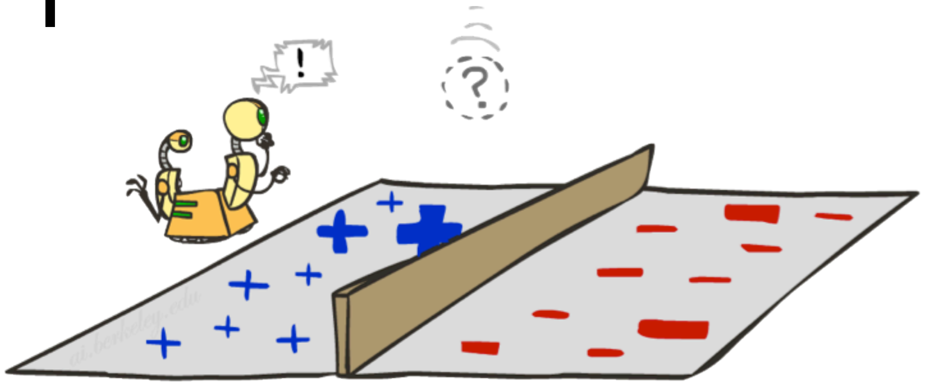



$$f \cdot w = 0$$

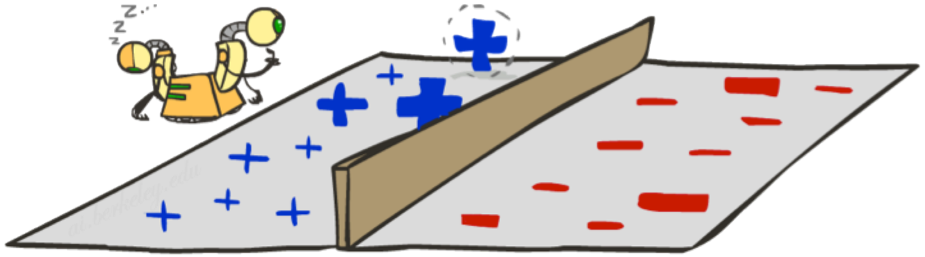
# Weight Updates

# Learning: Binary Perceptron

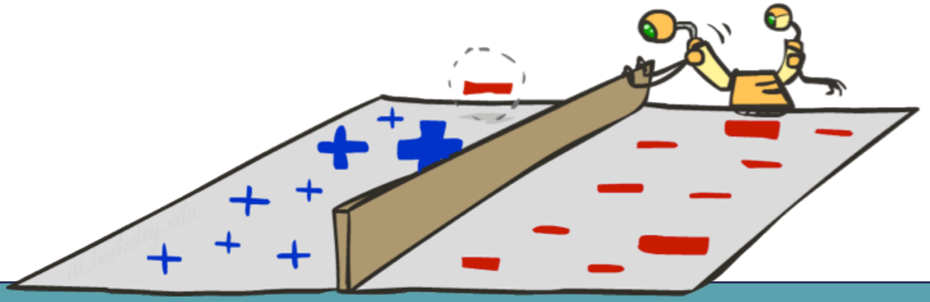
- Start with weights = 0
- For each training instance:
  - Classify with current weights



  - If correct (i.e., y=y*), no change!

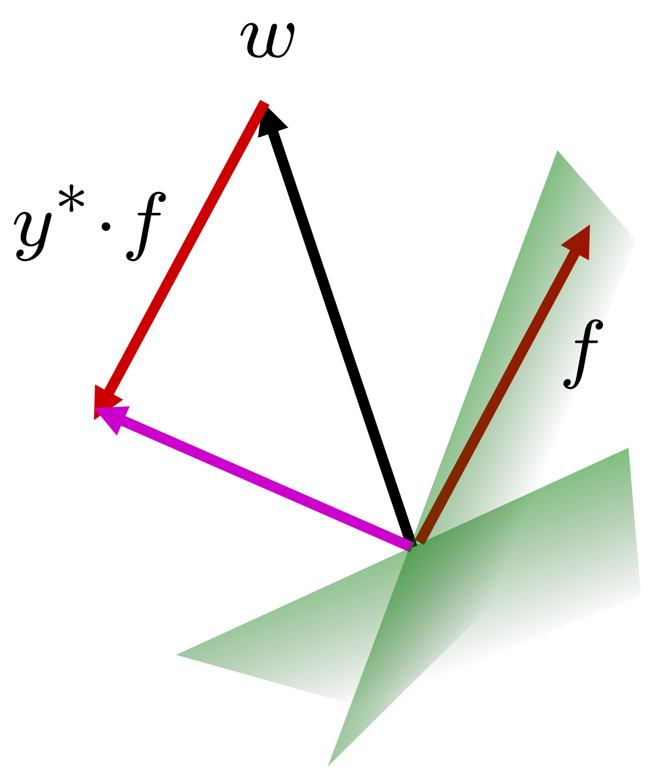


  - If wrong: adjust the weight vector



# Learning a Binary Perceptron

- Start with weights = 0
- For each training instance:
  - Classify with current weights
  - $y = \begin{cases} +1 & \text{if } \boldsymbol{w} \cdot \boldsymbol{f}(\boldsymbol{x}) \leq 0 \\ -1 & \text{if } \boldsymbol{w} \cdot \boldsymbol{f}(\boldsymbol{x}) > 0 \end{cases}$
  - If correct (i.e., y=y*), no change!
  - If wrong: adjust the weight vector by adding or subtracting the feature vector. Subtract if y* is -1.
  - $\boldsymbol{w} = \boldsymbol{w} + \boldsymbol{y}^* \cdot \boldsymbol{f}$



# Multiclass Decision Rule

- If we have multiple classes:
  - A weight vector for each class:

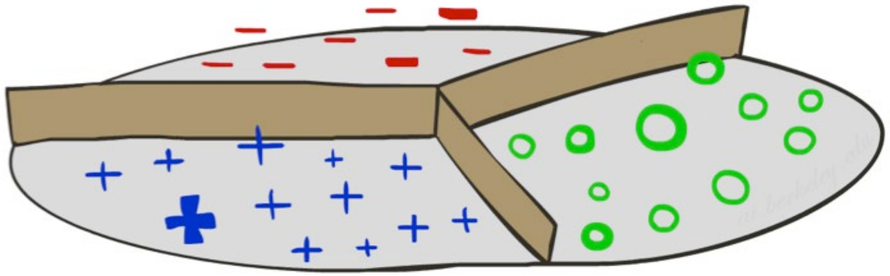
    $$w_y$$

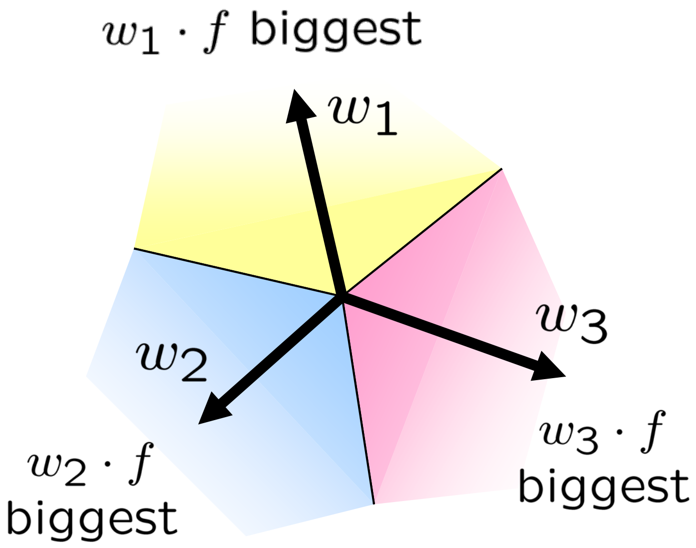
  - Score (activation) of a class y:

    $$w_y \cdot f(x)$$

  - Prediction highest score wins

    $$y = \arg\max_y \; w_y \cdot f(x)$$






*Binary = multiclass where the negative class has weight zero*

# Learning: Multiclass Perceptron

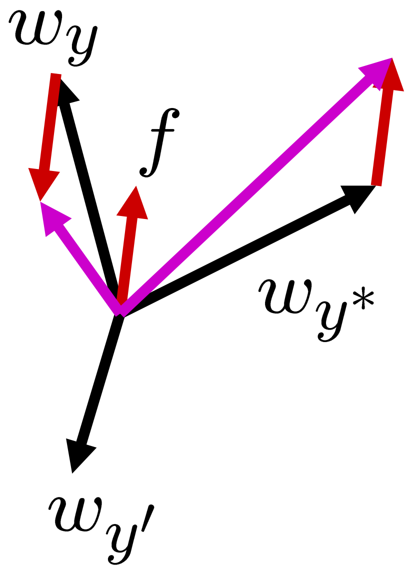
- Start with all weights = 0
- Pick up training examples one by one
- Predict with current weights

$$y = \arg\max_y \ w_y \cdot f(x)$$

$$w_y = w_y - f(x)$$

- If correct, no change!
- If wrong: lower score of wrong answer, raise score of right answer

$$w_{y^*} = w_{y^*} + f(x)$$



# Example: Multiclass Perceptron

"win the vote"

"win the election"

"win the game"

$w_{SPORTS}$

```
BIAS  : 1
win   : 0
game  : 0
vote  : 0
the   : 0
...
```

$w_{POLITICS}$

```
BIAS  : 0
win   : 0
game  : 0
vote  : 0
the   : 0
...
```

$w_{TECH}$

```
BIAS  : 0
win   : 0
game  : 0
vote  : 0
the   : 0
...
```

# Properties of Perceptrons

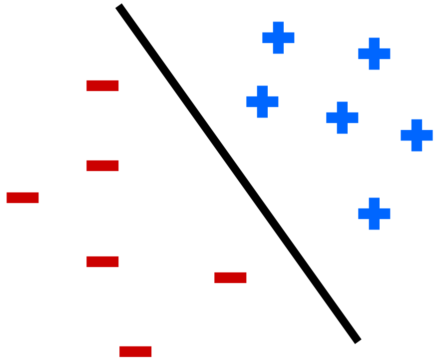
- Separability: true if some parameters get the training set perfectly correct

- Convergence: if the training is separable, perceptron will eventually converge (binary case)

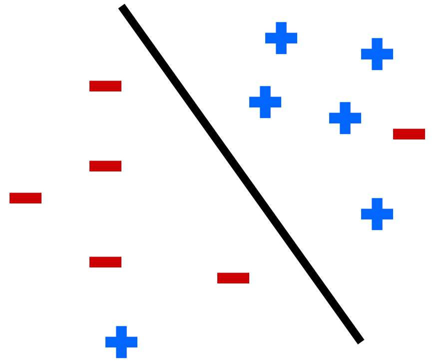
- Mistake Bound: the maximum number of mistakes (binary case) related to the *margin* or degree of separability

$$\text{mistakes} < \frac{k}{\delta^2}$$

Separable

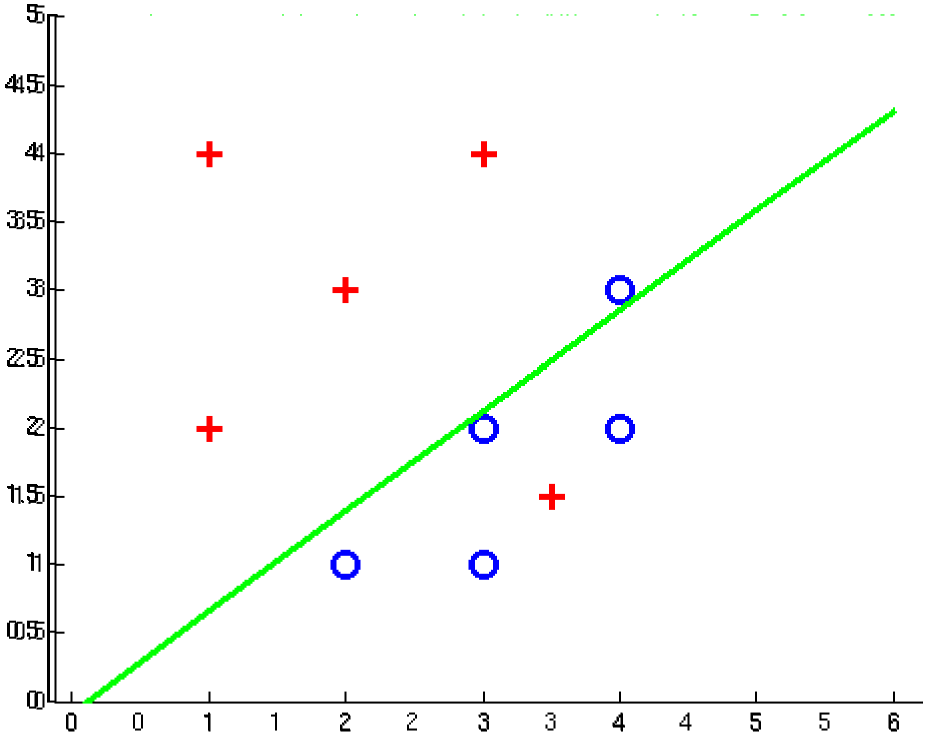


Non-Separable

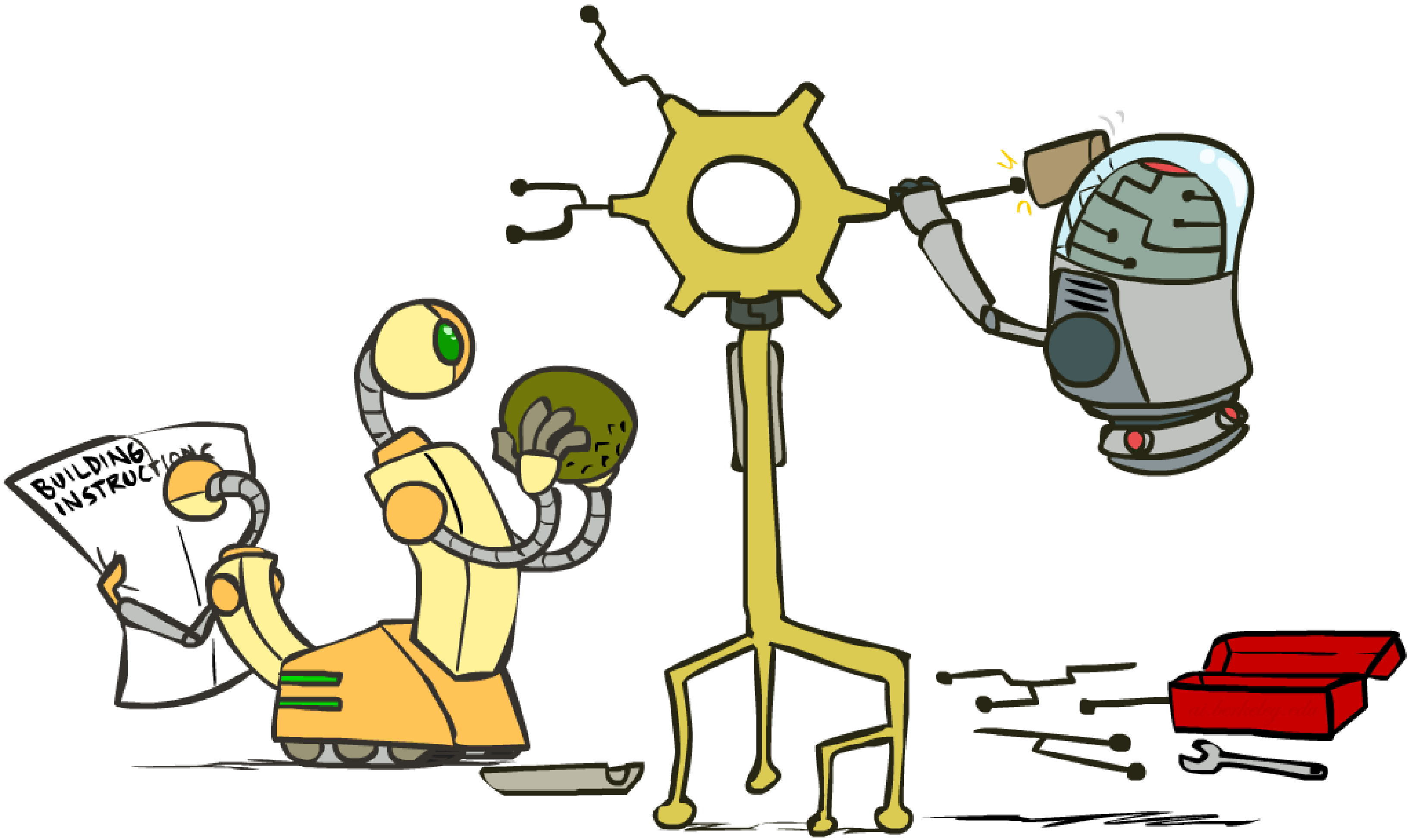


# Examples: Perceptron

- Non-Separable Case

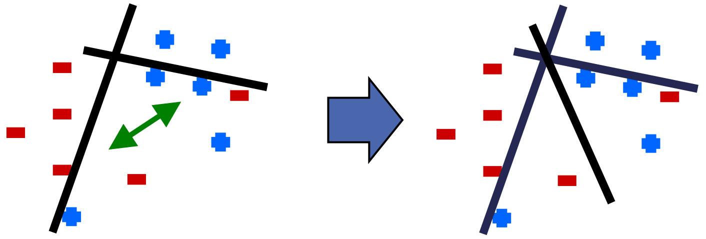


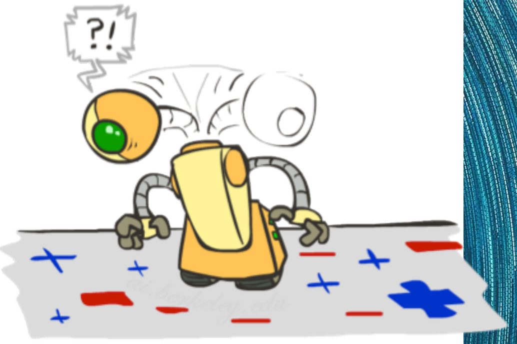
# Improving the Perceptron



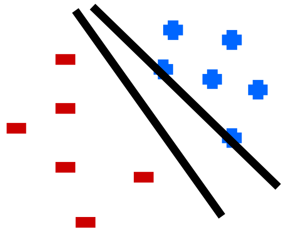
# Problems with the Perceptron

- Noise: if the data isn't separable, weights might thrash
  - Averaging weight vectors over time can help (averaged perceptron)

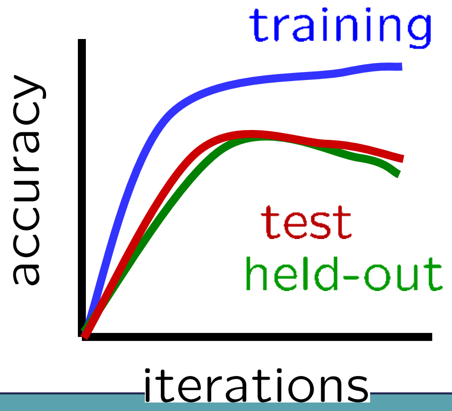


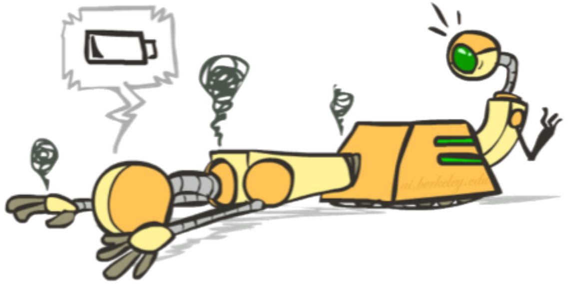



- Mediocre generalization: finds a "barely" separating solution



- Overtraining: test / held-out accuracy usually rises, then falls
  - Overtraining is a kind of overfitting



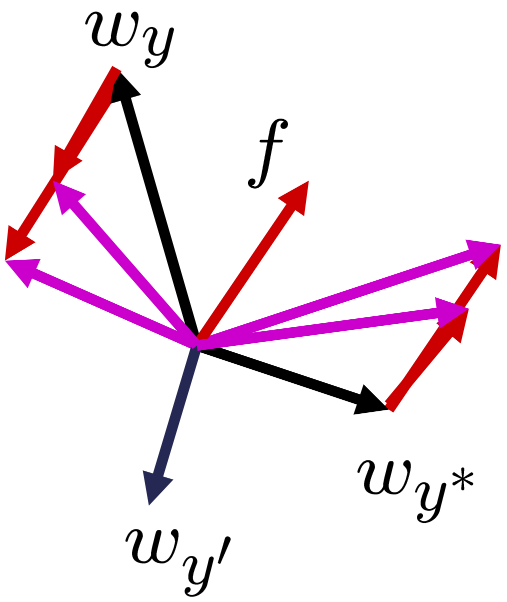
# Fixing the Perceptron

- Idea: adjust the weight update to mitigate these effects

- MIRA = Margin Infused Relaxed Algorithm
- Choose an update size that fixes the current mistake…
- … but, minimizes the change to w

$$\tau = \frac{(w'_y - w'_{y^*}) \cdot f + 1}{2f \cdot f}$$

The +1 helps to generalize




Guessed $y$ instead of $y^*$ on example $x$ with features $f(x)$

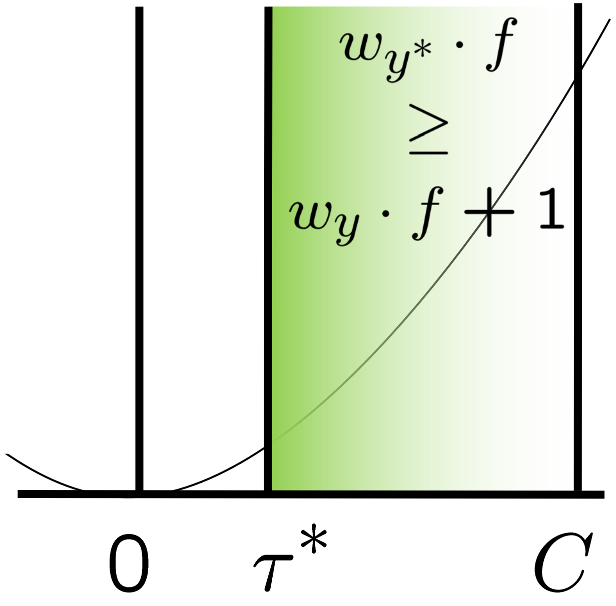
$$w_y = w'_y - \tau f(x)$$
$$w_{y^*} = w'_{y^*} + \tau f(x)$$

# Maximum Step Size

- In practice, it's also bad to make updates that are too large
  - Example may be labeled incorrectly
  - You may not have enough features
  - Solution: cap the maximum possible value of $\tau$ with some constant C

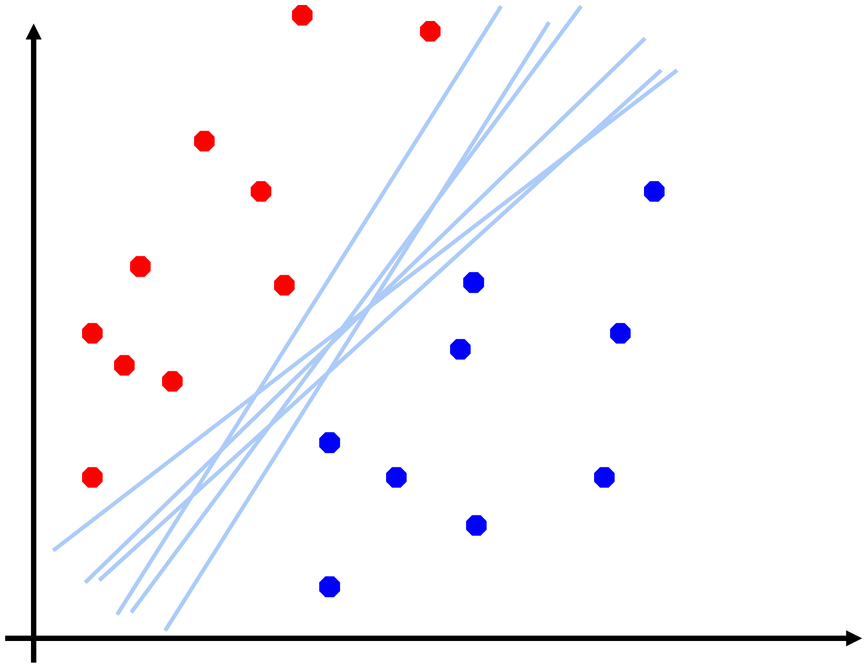
$$\tau^* = \min\left(\frac{(w'_y - w'_{y^*}) \cdot f + 1}{2f \cdot f}, C\right)$$

  - Corresponds to an optimization that assumes non-separable data
  - Usually converges faster than perceptron
  - Usually better, especially on noisy data



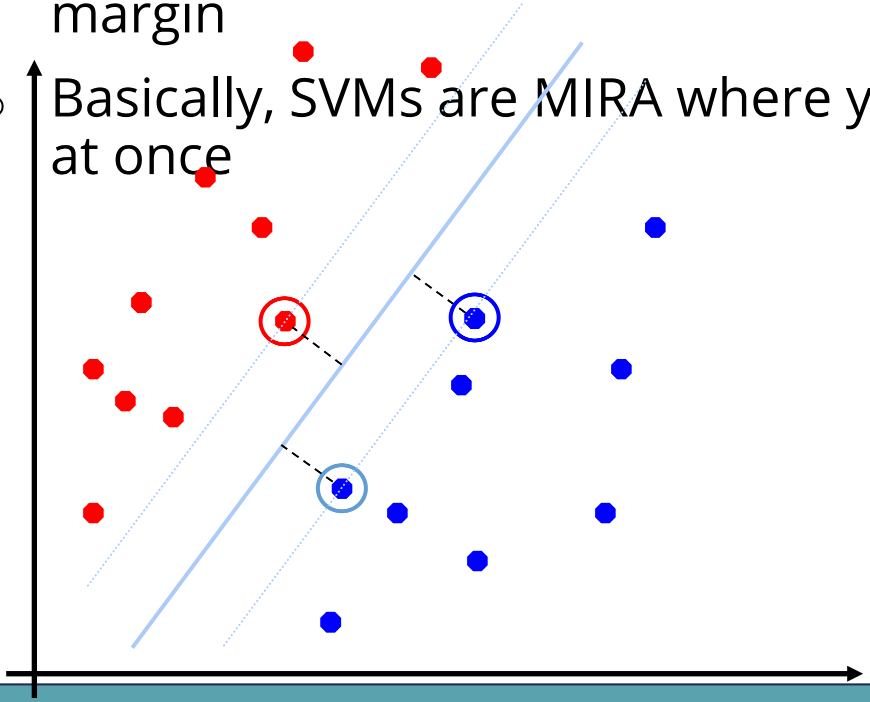
# Linear Separators

- Which of these linear separators is optimal?



# Support Vector Machines

- Maximizing the margin: good according to intuition, theory, practice
- Only support vectors matter; other training examples are ignorable
- Support vector machines (SVMs) find the separator with max margin
- Basically, SVMs are MIRA where you optimize ove
  at once



MIRA

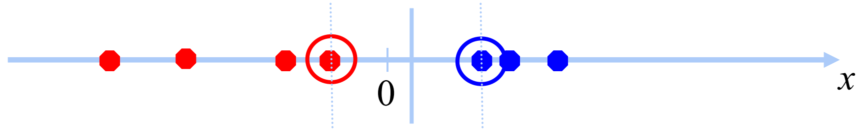
$$\min_w \frac{1}{2}||w - w'||^2$$
$$w_{y^*} \cdot f(x_i) \geq w_y \cdot f(x_i) + 1$$

SVM

$$\min_w \frac{1}{2}||w||^2$$
$$\forall i, y \; w_{y^*} \cdot f(x_i) \geq w_y \cdot f(x_i) + 1$$

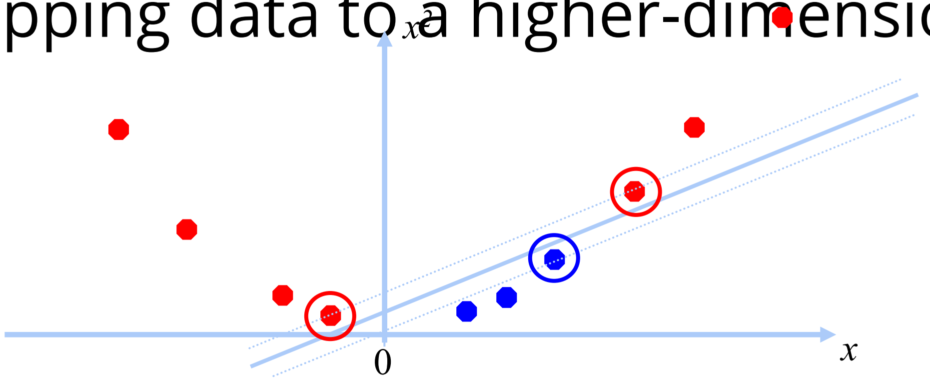
# Non-Linear Separators

- Data that is linearly separable works out great for linear decision rules:




- But what are we going to do if the dataset is just too hard?


- How about… mapping data to a higher-dimensional space:

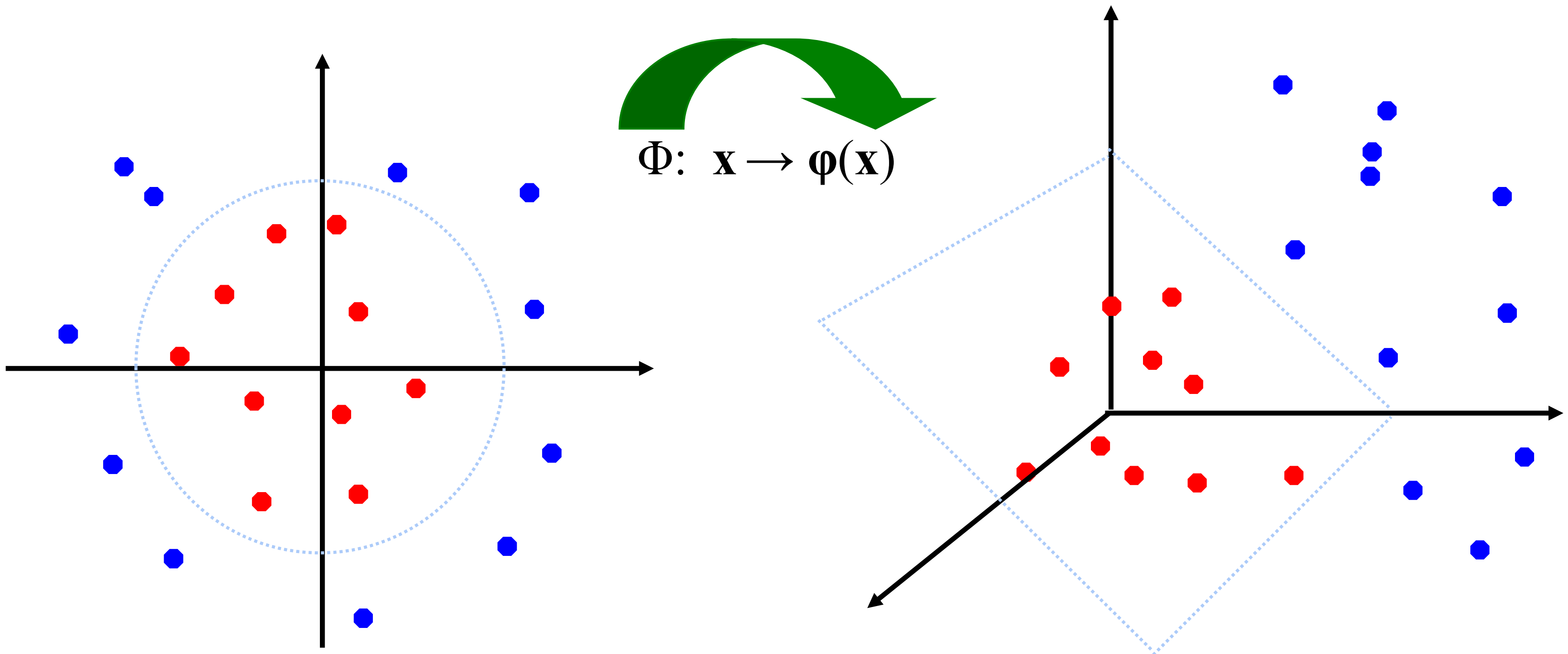





This and next few slides adapted from Ray Mooney, UT

# Non-Linear Separators

- General idea: the original feature space can always be mapped to some higher-dimensional feature space where the training set is separable:



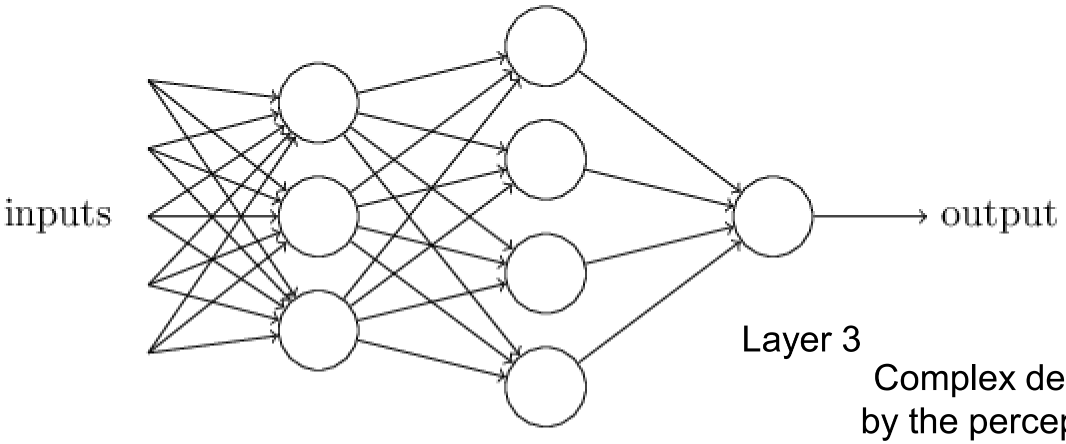
# Classification: Comparison

- Naïve Bayes
  - Builds a model training data
  - Gives prediction probabilities
  - Strong assumptions about feature independence
  - One pass through data (counting)

- Perceptrons / MIRA:
  - Makes less assumptions about data
  - Mistake-driven learning
  - Multiple passes through data (prediction)
  - Often more accurate

# Perceptrons for decision making

- A complex network of perceptrons could make quite subtle decisions:




Layer 3

Complex decisions can be made
by the perceptron in the third layer

Layer 1

Layer 2

1st layer makes 3 simple
decisions
by weighing the input evidence

2nd layer makes a decision by
weighing up the results from the
1st layer of decision-making

# Weights, bias and dot products

- Two notational changes simplify the way that perceptrons are described.
- The first change is to replace the weighted sum as a dot product

$$w \cdot x \equiv \sum_j w_j x_j$$

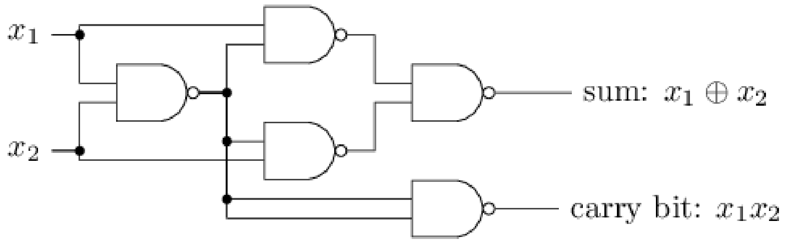
- The second change is to move the threshold to the other side of the inequality, and to replace it by a *bias*, b≡−threshold

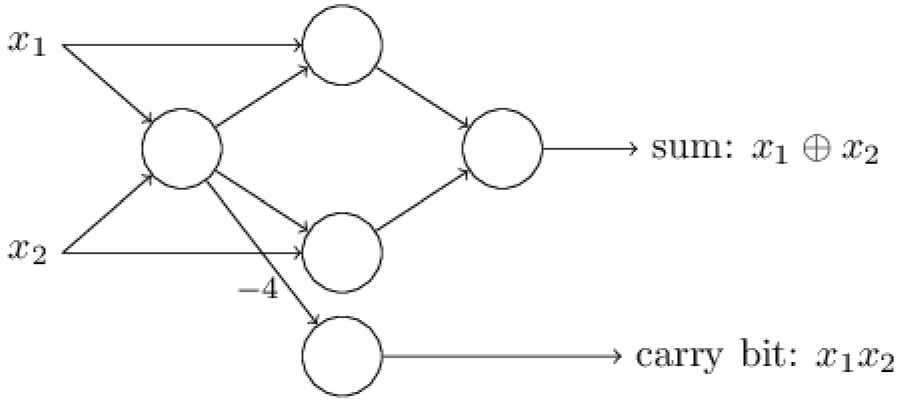
$$\text{output} = \begin{cases} 0 & \text{if } \sum_j w_j x_j \le \text{threshold} \\ 1 & \text{if } \sum_j w_j x_j > \text{threshold} \end{cases}$$

$$\text{output} = \begin{cases} 0 & \text{if } w \cdot x + b \le 0 \\ 1 & \text{if } w \cdot x + b > 0 \end{cases}$$

# Logical functions

- Networks of perceptrons to compute *any* logical function
- We can build any computation up out of NAND gates.
- For example, a circuit which adds two bits $x_1$ and $x_2$




All unlabeled weights are -2, all biases =3.

# The XOR problem

- A single neural unit cannot be used to compute the XOR function

**AND**

| x1 | x2 | y |
|---|---|---|
| 0 | 0 | 0 |
| 0 | 1 | 0 |
| 1 | 0 | 0 |
| 1 | 1 | 1 |

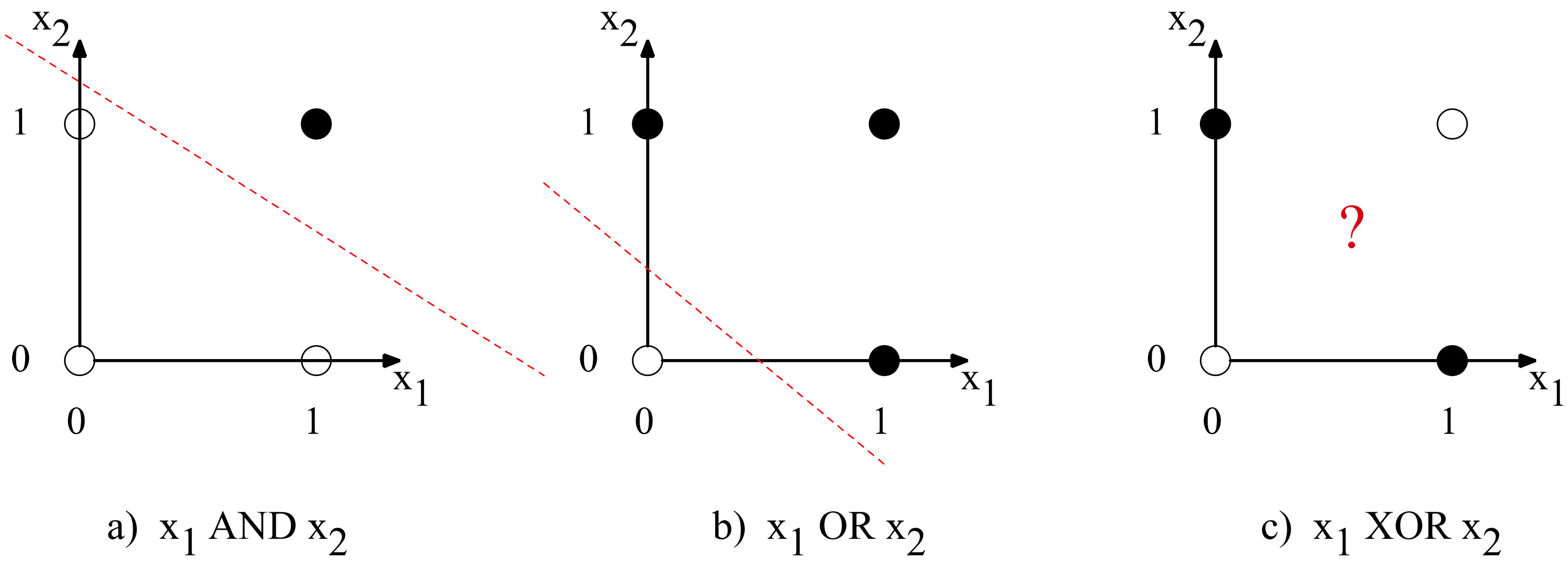
**OR**

| x1 | x2 | y |
|---|---|---|
| 0 | 0 | 0 |
| 0 | 1 | 1 |
| 1 | 0 | 1 |
| 1 | 1 | 1 |

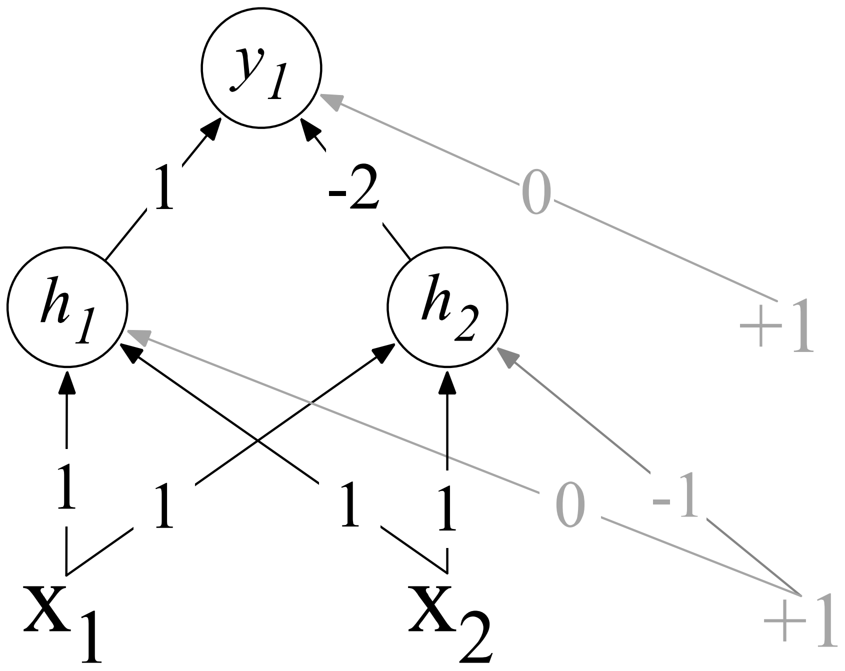
**XOR**

| x1 | x2 | y |
|---|---|---|
| 0 | 0 | 0 |
| 0 | 1 | 1 |
| 1 | 0 | 1 |
| 1 | 1 | 0 |

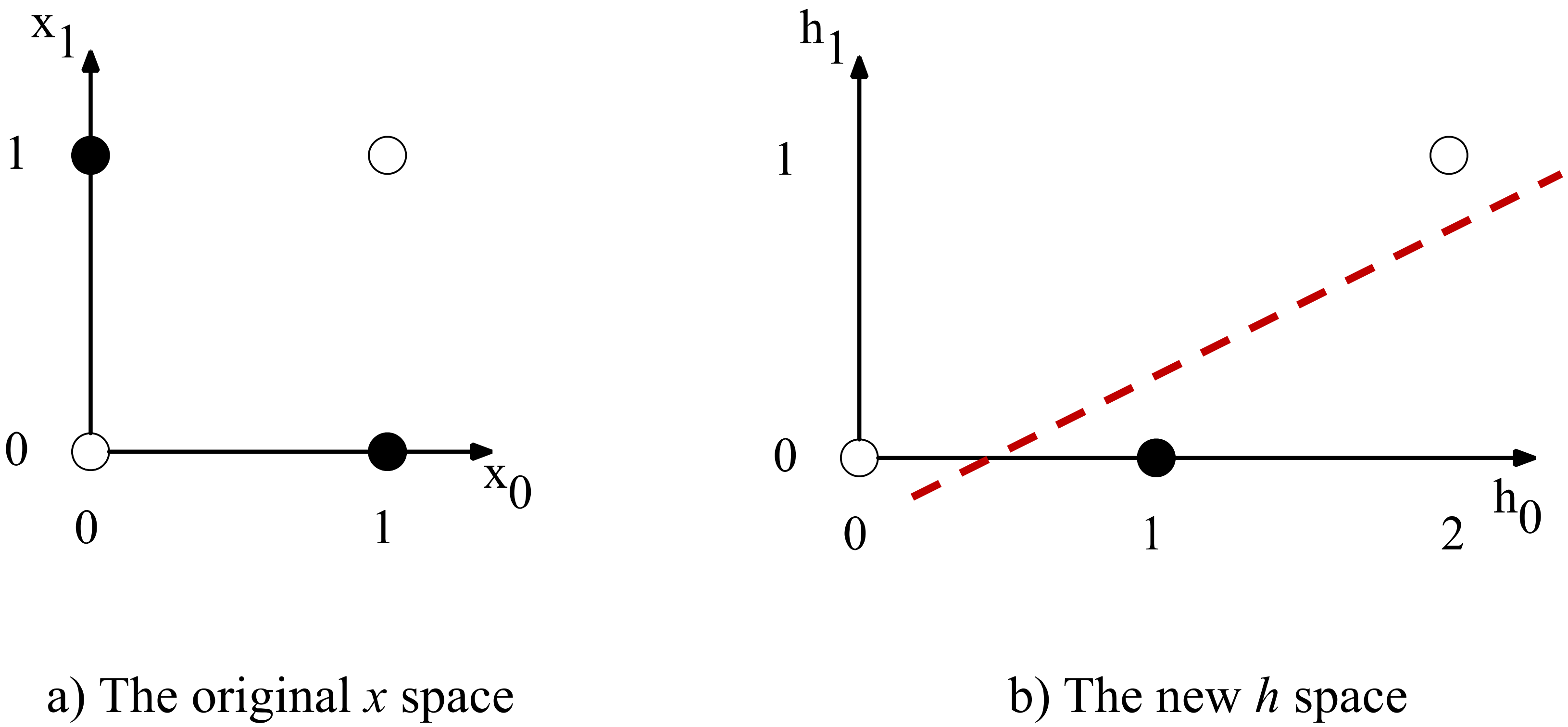
# The XOR Problem



# The XOR Solution

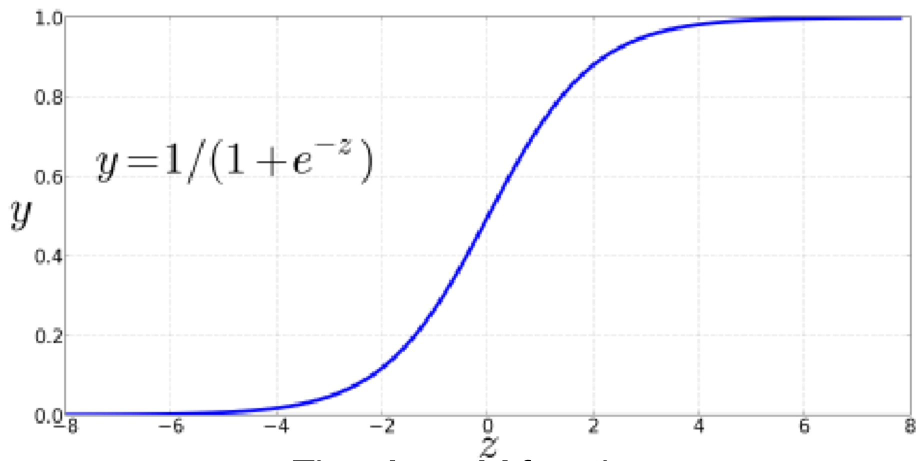


# The XOR Solution



# Activation Functions

- Instead of directly outputting $z = w \cdot x + b$, which is a linear function of $x$, neuron units apply a non-linear function $f$ to $z$.
- The output of this function is called the **activation value** for the unit, represented by the variable **a**. The output of a neural network is called **y,** so if the activation of a node is the final output of a network then
  - $y = a = f(z)$
- There are 3 commonly used non-linear functions used for $f$:
  The **sigmoid** function
  The **tanh** function
  The **rectified linear unit ReLU**




The **sigmoid** function

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